# HEAT TRANSFER FOR FORCED FLOW OF PHASE MIXTURE IN A PIPE 

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#### Abstract

An experimental investigation of heat transfer for flow of a two-phase water-ice mixture in a pipe is performed. A model of heat transfer is proposed, and the obtained experimental data are generalized.


The processes of motion of two-phase mixtures with a dispersed solid phase through pipelines are widespread in modern technology. In some cases, for example, in refrigeration engineering, a two-phase frazil water-ice mixture is considered as a promising heat transfer agent [1]. Simple qualitative considerations suggest that in the process of motion of such a medium along the heat-transfer surface on account of melting of ice particles highly efficient heat transfer (as compared to water flow) is to be observed, which seems promising in creating new kinds of heat transfer equipment. In this connection we performed a theoretical-experimental investigation of heat transfer in flow of a model water-ice mixture (water frazil) in pipes of varying diameter.

The investigations were performed on the experimental plant shown in Fig. 1. The section of a transparent pipe with a coil-to-coil wound copper wire 0.1 mm in diameter served as a heat transfer sensor. The wire was laid in the inner surface of the pipe. The wire ends were connected to a bridge circuit in such a way that the sensor was one of the arms of the resistance bridge. An F-30 type measuring voltmeter was placed across a diagonally opposite pair of junctions of the bridge, which was powered by an independent power source. The sensor length was 100 mm .

To produce frozen water droplets use was made of the method of vacuum freezing in the process of free fall of the liquid droplets in a column 2 mm in height. Such a method enabled us to produce practically spherical particles of ice, which are formed through intense evaporation of water droplets. A necessary condition for the realization of such a method is maintaining a pressure in the column lower than that of water vapor at the triple point ( 615 Pa ).

Thus produced frozen water droplets were mixed in a thermostat with water at a temperature of about $0^{\circ} \mathrm{C}$. A mixture of the frozen droplets and the water was pumped from the thermostat through a pipe using a centrifugal pump through the experimental section. To maintain the prescribed concentration of the solid phase, a cooler, which was a spiral tube with perforated walls, was placed into the thermostat.

Under the action of the pressure of the vapor formed on the heater by heat release, liquid nitrogen from a Dewar vessel flowed through the cooler, bubbled through the layer of water, cooling it. By mixing the liquid the pump uniformly distributed the frozen droplets in the water, which were introduced in the thermostat via a special device, enabling us to meter out the amount of particles. The mixture temperature $\mathrm{T}_{\text {mix }}$ was taken by Chromel-Coppel thermocouples and was maintained constant automatically by using the cooler, the contact thermometer, and the switching relay.

The average ice particle diameter, which varied from 1 to 15 mm in the experiments, was determined by direct measurement in a photograph of the flow pattern. The value of the heat transfer coefficient was determined by the formula

$$
\begin{equation*}
\alpha=Q / F\left(T_{w}-T_{\operatorname{mix}}\right), \tag{1}
\end{equation*}
$$

where Q is the quantity of heat released on the sensor; F is the sensor surface area. The temperature difference $\mathrm{T}_{\mathrm{w}^{-}}$ $T_{\text {mix }}$ was maintained at $7-10^{\circ} \mathrm{C}$.

To compare the values of the heat transfer coefficients for various flowing media, heat transfer of a single-
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Fig. 1. Diagram of the experimental plant: 1) Dewar vessel; 2) heater; 3) switching relay; 4) contact thermometer; 5) hydrodynamic channel; 6) heat transfer sensor; 7) electrical bridge; 8) flowmeter; 9) thermostat; 10) centrifugal pump; 11) cooler.
phase flow (water) with different hydrodynamic flow regimes was also investigated. Experimental data were generalized in the form of standard dependences $\mathrm{Nu}=\mathrm{f}(\mathrm{Re}, \mathrm{Pr}$ ) (see Fig. 2). For water in the range $3000 \leq \operatorname{Re} \leq$ 80000 we obtained

$$
\begin{equation*}
\mathrm{Nu}_{q}=0,023 \mathrm{Re}^{0,8} \mathrm{Pr}_{f}^{0,43}\left(\operatorname{Pr}_{f} / \operatorname{Pr}_{w}\right)^{0,25}, \tag{2}
\end{equation*}
$$

which is in good agreement with the known Mikheev formula [2].
For water-ice flows with the ice concentration 0.2 and 0.3 we obtained respectively

$$
\begin{array}{ll}
\mathrm{Nu}=0,095 \mathrm{Re}^{0,8} \operatorname{Pr}_{f}^{0,43}\left(\operatorname{Pr}_{f} / \operatorname{Pr}_{w}\right)^{0,25} & \left(\mathrm{Re}=3 \cdot 10^{3} \div 3 \cdot 10^{4}\right), \\
\mathrm{Nu}=0,201 \mathrm{Re}^{0,8} \mathrm{Pr}_{f}^{0,43}\left(\operatorname{Pr}_{f} / \operatorname{Pr}_{w}\right)^{0,25} & \left(\mathrm{Re}=3 \cdot 10^{3} \div 1,4 \cdot 10^{4}\right) . \tag{4}
\end{array}
$$

A formal generalization of the experimental data for different values of porosity resulted in formulating the equation

$$
\begin{equation*}
\left(\mathrm{Nu}-\mathrm{Nu}_{0}\right) / \mathrm{Nu}_{0}=126,8(1-\varepsilon)^{2,3} \quad(1-\varepsilon=0 \div 0,3), \tag{5}
\end{equation*}
$$

where $\mathrm{Nu}_{0}$ is calculated by formula (2).
To analyze the process of heat transfer of the investigated system in greater detail, we applied a simple phenomenological model of heat transfer. It is based on the following assumptions. At any instant the portion of the heat transfer surface $A^{*} S$ makes contact with ice particles, and the other one ( $1-\mathrm{A}^{*}$ ) S - with water. The heat transfer coefficient in the 1st zone is $\alpha_{\mathrm{s}}$, and in the second $\alpha_{0}$, which is calculated by formula (2). As an approximation use is made of the following representation for the coefficient $\mathrm{A}^{*}: \mathrm{A}^{*}=\mathrm{A}(1-\varepsilon)$. The process of heat transfer from the pipe wall to the ice particles, as in an ordinary dispersed layer [3], is stationary merely in the statistical sense. It is realized by numerous nonstationary acts of heat transfer of individual particles with the surface.

The averaged heat transfer coefficient of the two-phase water-ice medium within the framework of this scheme can be represented in the following dimensionless form:

$$
\begin{equation*}
\mathrm{Nu}=A(1-\varepsilon) \mathrm{Nu}_{s}+(1-A(1-\varepsilon)) \mathrm{Nu}_{0}^{*)} . \tag{6}
\end{equation*}
$$

Within the framework of (6) determining Nu reduces to the calculation of $N u_{s}$ and $\mathrm{A}\left(\mathrm{Nu}_{0}\right.$ is calculated from (2)).

[^0]

Fig. 2. Heat transfer for flow of water frazil in tubes with different diameters ( N $\left.\left.=\mathrm{Nu} / \operatorname{Pr}_{\mathrm{f}}^{0.43}\left(\operatorname{Pr}_{\mathrm{f}} / \operatorname{Pr}_{\mathrm{w}}\right)^{0.25}\right): 1,2,3\right) \mathrm{d}=25,30$, and 40 mm respectively; I) water; II, III) frazil with the ice concentration in water 0.2 and 0.3 respectively.

To determine $\mathrm{Nu}_{\mathrm{s}}$, we will consider the problem of thawing of ice in contact with a flat wall, having the temperature $\mathrm{T}_{\mathrm{w}}$. The mathematical formulation of the problem has the form, assuming that the ice temperature is $\mathrm{T}_{\mathrm{s}}$ $=\mathrm{T}_{\mathrm{fr}}$ :

$$
\begin{equation*}
\partial T / \partial t=a_{f} \partial^{2} T / \partial x^{2} \tag{7}
\end{equation*}
$$

with the boundary conditions

$$
\begin{gather*}
T(\xi, t)=T_{f r}, T(0, t)=T_{w}, \\
\lambda_{f} \frac{\partial T_{j}}{\partial x}=-\left.\varepsilon^{*} \rho_{s} \frac{d \xi}{d t}\right|_{x=\xi} . \tag{8}
\end{gather*}
$$

The solution (7), (8) has the form [4]:

$$
\begin{equation*}
\theta=\left(T_{f}-T_{w}\right) /\left(T_{f r}-T_{w}\right)=\operatorname{erf} \frac{x}{2 \sqrt{t a_{f}}} / \operatorname{erf} \frac{\beta}{2 \sqrt{a_{f}}} \tag{9}
\end{equation*}
$$

where the coefficient $\beta$ is determined from the solution of the characteristic equation

$$
\begin{equation*}
\lambda_{f}\left(T_{w}-T_{f r}\right) \exp \left(-\beta^{2} / 4 a_{f}\right) / \sqrt{a_{f}} \operatorname{erf}\left(\beta / 2 \sqrt{a_{f}}\right)=\mathrm{e}^{*} \rho_{s} \sqrt{\pi} \beta / 2 \tag{10}
\end{equation*}
$$

For the conditions of the performed experiments $\left(\mathrm{T}_{\mathrm{w}}-\mathrm{T}_{\mathrm{fr}} \sim 7-10^{\circ} \mathrm{C}\right.$ ) $\beta \sim 1.55 \cdot 10^{-4}$.
The heat transfer coefficient $\alpha_{s}^{*}$ is determined as

$$
\begin{equation*}
\alpha_{s}^{*}=-\left.\frac{\lambda_{f}}{T_{w}-T_{f r}} \frac{\partial T_{j}}{\partial x}\right|_{x=0} \tag{11}
\end{equation*}
$$

On the basis of formula (8) for $\alpha_{\mathrm{s}}^{*}$ we obtain

$$
\begin{equation*}
\alpha_{s}^{*}(t)=\lambda_{f} / \sqrt{\pi t a_{f}} \operatorname{erf}\left(\beta / 2 \sqrt{a_{f}}\right) . \tag{12}
\end{equation*}
$$

The average (over the contact time $\tau$ ) value of $\alpha_{\mathrm{s}}^{*}$ will be equal to

$$
\begin{equation*}
\left\langle\alpha_{\mathrm{s}}^{*}\right\rangle=\frac{1}{\tau} \int_{0}^{\tau} \alpha_{\mathrm{s}}^{*}(t) d t=2 \lambda_{f} / \sqrt{\pi \tau a_{f}} \operatorname{erf}\left(\beta / 2 \sqrt{a_{f}}\right) . \tag{13}
\end{equation*}
$$

For the sought coefficient $\alpha_{\mathrm{s}}$, entering in (6), we have the standard representation in terms of the distribution function for the values of $\tau$ :

$$
\begin{equation*}
\alpha_{s}=\int_{0}^{\infty}\left\langle\alpha_{s}^{*}\right\rangle f(\tau) d \tau . \tag{14}
\end{equation*}
$$

As a first approximation, we will define $\alpha_{\mathrm{s}}$ in terms of the average duration of stay of the ice particles near the heat transfer surface $\langle\tau\rangle$ :

$$
\begin{equation*}
\alpha_{s}=2 \lambda_{f} / \sqrt{\pi\langle\tau\rangle a_{f}} \operatorname{erf}\left(\beta / 2 \sqrt{a_{f}}\right) . \tag{13a}
\end{equation*}
$$

The parameters of a theoretical model A and $\langle\tau\rangle$ are determined in the standard way - by comparison with the experimental data on the values of $\alpha$. For $\langle\mathrm{Fo}\rangle=\mathrm{a}_{\mathrm{f}}\langle\tau\rangle / \mathrm{d}^{2}$ use was made of the following power dependence:

$$
\begin{equation*}
\langle\mathrm{F} 0\rangle=B(1-\varepsilon)^{-n_{1}} \mathrm{Re}^{-n_{\mathrm{x}}} . \tag{15}
\end{equation*}
$$

The value of $n_{2}$ can easily be determined from comparing expressions (5) and (6), taking into account the independence of the coefficient A of $\mathrm{Re}:\langle\mathrm{Fo}\rangle \sim \mathrm{Re}^{-1.6}$, i.e., $\mathrm{n}_{2}=1.6$. To determine the three coefficients $\mathrm{A}, \mathrm{B}$, and $\mathrm{n}_{1}$, use is made of the system of equations:

$$
\begin{equation*}
A\left(1-\varepsilon_{i}\right) \mathrm{Nu}_{s_{i}} / \mathrm{Nu}_{0}+\left(1-A\left(1-\varepsilon_{i}\right)\right)=126,8\left(1-\varepsilon_{i}\right)^{2,3}+1, \quad i=1,2,3, \tag{16}
\end{equation*}
$$

where $\mathrm{Nu}_{\mathrm{s}_{\mathrm{i}}}=\alpha_{\mathrm{s}_{\mathrm{i}}} \mathrm{d} / \lambda_{\mathrm{f}}$ are calculated from (13a) and (15). A numerical solution of the system (16) yielded the following values of the coefficients: $\mathrm{A}=1.92, \mathrm{~B}=1.29, \mathrm{n}_{1}=2.20$.

As an example we will perform a concrete calculation of the average time of contact of ice particles with the heat transfer surface under the following conditions: $\varepsilon=0.7 ; \mathrm{Re}=10^{4} ; \mathrm{d}=0.04 \mathrm{~m} .\langle\tau\rangle=\langle\mathrm{Fo}\rangle \mathrm{d}^{2} / \mathrm{a}_{\mathrm{f}}=$ $1.29(1-\varepsilon)^{2.20} \mathrm{Re}^{-1.6} \mathrm{~d}^{2} / \mathrm{a}_{\mathrm{f}} \approx 0.086 \mathrm{sec}$, which corresponds to the frequency of change of the ice particles near the surface $\approx 11.61 / \mathrm{sec}$. It is significant that the obtained value of the coefficient $\mathrm{A}=1.92$ indicates the increased concentration of the ice near the pipe walls. It should be expected that the value of the heat transfer coefficient of the frazil liquid (water-ice mixture) is dependent on the wettability conditions of the heat transfer surface, since it is on wettability that the values of the coefficients A and B depend. Unfortunately, we did not investigate the influence of this factor.

As the present work shows, the use of a frazil liquid as a heat transfer agent enables us to significantly intensify the process of heat transfer.

The proposed phenomenological model makes it possible to generalize the obtained experimental data and to satisfactorily explain the substantial increase in the heat transfer coefficient as the concentration of the ice particles in water increases. This is due to the ice-water phase transition and the stability of the temperature difference between the heat transfer surface and the moving boundary of thawing of the ice caused by this.

## NOTATION

a, thermal diffusivity coefficient; A, B, dimensionless coefficients; d, diameter of pipe; c , specific heat; Nu $=\alpha \mathrm{d} / \lambda_{\mathrm{f}}$, Nusselt number; $\operatorname{Pr}=\mathrm{c}_{\mathrm{f}} \eta_{\mathrm{f}} / \lambda_{\mathrm{f}}$, Prandtl number; $\operatorname{Re}=u d \rho_{\mathrm{f}} / \eta_{\mathrm{f}}$, Reynolds number; $\mathfrak{t}$, time; T, temperature; $\mathbf{x}$, coordinate; $\alpha$, heat transfer coefficient; $\rho$, density; $\eta$, dynamic viscosity; $\lambda$, thermal conductivity; $\varepsilon$, volume-mean porosity; $\varepsilon^{*}$, phase transition heat; $\mathbf{x}=\xi$, boundary of melt zone. Subscripts: f , water; fr, freezing point; s , ice; $\mathbf{w}$, heat transfer surface.

## REFERENCES

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[^0]:    ${ }^{*}$ In writting this relation, in fact, it is assumed that the liquid temperature in the flow core is equal to the melting temperature of the ice.

